The Informational Role of Stock Prices and the Macroeconomy *

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Abstract

A theoretical and empirical literature in financial economics shows that stock prices provide useful information to firms which act under imperfect information. I take a macroeconomic perspective on the subject to study how an aggregate shock impacts the informativeness of stock prices and in turn affects the degree of input misallocation in the economy. I show that the interaction between the real and the financial side of the economy gives rise to an amplification mechanism. As firms grow in an expansion, their profits are more exposed to the realization of their fundamental. Speculators, which can acquire private information on it, will do more so as the rent they can extract is larger. The stock price becomes more informative about the fundamental, the informational friction is alleviated and output gets closer to the perfect information benchmark. The amplification mechanism is novel in that it links together input misallocation and the informational role of stock prices.

Key-words: misallocation, imperfect information, informative financial markets, aggregate shocks, amplification mechanism.

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1 Introduction

A major narrative about the informational role of stock markets is the following (Bond, Edmans, and Goldstein, 2012). Firms do not perfectly observe some variable concerning their market activity, call it "the fundamental" (e.g. their productivity level, their demand, the positions of competitors, etc.), which prevents them from efficiently allocating factors to production. For example, they could overestimate their productivity and demand an amount of inputs that is greater than the profit-maximizing one. However, firms know that stock prices are good predictors of their performance. The reason is that traders are investors that have a facilitated access to information about the profitability of firms and stock prices partially reflect it. Therefore, when a firm observes a high price, for example, she expects her fundamental also to be high and so allocates a big amount of factors to production. In this sense, *the stock market produces information* that is useful to the firms' input choices. The higher the correlation between the price and the fundamental, the more informative the price and so the less severe the informational friction.

The aim of my work is to give a macroeconomic perspective to this story of input decisions under imperfect information and informative stock markets. More specifically, I study how the production of information by stock markets can vary over the business cycle and so reduce or magnify the informational friction affecting firms' input decisions, with consequences on the degree of misallocation in the economy and so on aggregate efficiency and output.

As far as I know, this paper is one of the few bringing together the informational role of financial markets and input misallocation. The reason is that, on one side, the literature on the real effects of financial markets has primarily focused on a single firm, without exploring in depth the economy-wide implications (Dow, Goldstein, and Guembel (2017) is an exception from which, indeed, this paper takes inspiration). On the other side, the literature investigating the sources of input misallocation has devoted particular attention to adjustment costs (Asker, Collard-Wexler, and De Loecker, 2014) and financial frictions (Midrigan and Xu (2013), Moll (2014), Buera, Kaboski, and Shin (2011)) but not as much to the role of imperfect information and financial markets. Only recently, David, Hopenhayn, and Venkateswaran (2016) identifies imperfect information as a cause of misallocation and considers financial markets as institutions that can potentially reduce the informational friction. Differently from theirs, my contribution consists in studying the role of stock markets in mitigating informational frictions *at business cycle frequencies*, rather than *in the long-run*. In other words, I analyze how information production in the stock market can amplify aggregate shocks rather than how it permanently affects output. In this sense, the two theories can be thought of as complementary.

Very narrowly defined, my research question is: how does an aggregate shock affect information production in the stock market? How does this endogenous response of information production feedback into input misallocation and so into aggregate output? To provide an answer, I set-up a model where the aggregate state of the economy, input choices and information production are linked together. On one side, firms choose inputs under imperfect information about their productivity. On the other side, speculators in the stock market receive an informative signal on the firms' productivity and can choose its precision by paying a cost. In equilibrium the signal's precision trades off the benefits, consisting in higher expected trading profits, with the acquisition cost. Because the expected trading profits are endogenous to the trading strategy, which in turn depends on the way the stock is priced, that is linked to firm's input decisions, I model a game between privately-informed speculators trading the asset, a firm choosing inputs based on her prior belief on her productivity level and what she learns from the stock price and a market-maker pricing the stock. This allows me to find the expected trading profits for a given level of information precision and so derive the equilibrium level of information precision. An aggregate shock modifies the mean or the dispersion of the prior distribution of the firm-specific productivity and so affects firm decisions, which for the reasons described above impact the expected trading profits and so the equilibrium level of information precision. I find that a positive aggregate productivity shock is amplified by the endogenous response of information produced by the stock market. However, even when traders acquire perfectly informative signals, the informational friction is not completely removed because of their profit-seeking behaviour.

I illustrate the amplification mechanism. When an aggregate shock hits the economy, firms are more productive on average, which implies that they are more optimistic about their productivity level, in the absence of new information inferred from their own stock price. So, they demand a larger amount of inputs. As firms grow, their profits are more exposed to the realization of the fundamental: the gap between profits when productivity is high and when is low widens. On the side of the financial market, the consequence of having firm's profits fluctuating more between the high- and low-productivity case is that the (potential) mispricing of the stock increases. For example, if stocks are priced at the average profit of the firm, an aggregate shock that shifts profits of highly productive firms from 10 to 20 and leaves profits of low productive firms to 0 enlarges the mispricing from 5 to 10. As a result, traders are willing to acquire more precise private information to know which is the true productivity level.

Speculators trade more aggressively and so stock prices become more informative, the informational friction affecting firms' decisions is less severe, input misallocation decreases and aggregate output grows.

Related literature. My model studies how the business cycle impacts the ability of financial markets to provide useful information to firms who allocate factors under imperfect information. Therefore, on one side my work builds on the literature on informative financial markets and on the other side on the literature linking imperfect information to resource misallocation and hence to aggregate productivity and output. I review them in the presented order.

The theory of informative financial markets dates back to Grossman and Stiglitz (1980) and Kyle (1985) and has been extended in the later years (Albagli, Hellwig, and Tsyvinski (2013) Albagli, Hellwig, and Tsyvinski (2021)). Limits to arbitrage and the presence of speculators trading at random (noise or liquidity traders) prevent the equilibrium price to fully reveal the private information of traders. They are therefore willing to pay a cost to acquire a private signal as this allows them to identify the mispricing of the asset. As the price of the asset is a byproduct of trading activity, it is (imperfectly) informative about the underlying fundamental. I employ a market micro-structure that is in the spirit of Kyle (1985).

In my model stock prices not only are informative about the firms' fundamental, but exactly because they are so they affect the allocation of their resources and so have a real effect. More precisely, because information production is endogenous, the interaction between the real and financial side of the economy gives rise to a two-way feedback: shocks to the fundamental (real variable) modify the incentives of traders to acquire more precise signals (financial variable); this, in turn, varies the informativeness of the stock price and the firm decision on capital (real variable) changes as a consequence. Therefore this paper is in the tradition of the literature exploring the real effects of financial markets (see Bond, Edmans, and Goldstein (2012) for a survey on theoretical and empirical contributions on the subject).

My work significantly inherits from Dow, Goldstein, and Guembel (2017) which studies endogenous information production in a context where firms decide on a binary action, e.g. merging with another firm or not, under imperfect information of the outcome. Instead, I consider that firms' control variable is continuous (their level of capital), I give a clear counterpart to "firm value" as firm profits and I introduce several firms and aggregate shocks. This allows me to investigate how economy-wide shocks affect price informativeness and how such changes are transferred to the macroeconomy in the form of an increase or a reduction of the informational friction.

As with regards to the literature linking resource misallocation to imperfect information, my work is related to David, Hopenhayn, and Venkateswaran (2016). This paper analyzes the the macroeconomic consequences of resource misallocation due to informational frictions on the side of producers and empirically assess the contribution of stock prices to the reduction of the implied efficiency loss. The authors find that learning from financial markets little reduces residual uncertainty: it eliminates only 2% of fundamental uncertainty, which corresponds to TFP gains between 0.2% and 1.3%. Since they focus on long-run consequences rather than fluctuations driven by aggregate shocks, my theory can be considered as a complement rather than a substitute. The scarce contribution of financial markets to the firms' learning process in the long-run can be rationalized with my finding that in booms price informativeness rises and in recessions it drops.

The rest of the paper is organized as follows. Section 2 describes my model of production and financial market under imperfect information. Section 3 jointly analyzes the trading decisions of speculators, the the input choice of a firm and the pricing rule of a market-maker as the equilibrium of a game. It then derives the equilibrium level of information precision. Section 4 introduces the aggregate economy and illustrates the amplification mechanism. Section 5 concludes.

2 The model

In this section I develop a model of production and financial market activity under imperfect information. I first line out the production side of the economy and later I explain the micro-structure of the financial market where the firms' stocks are traded. Throughout the paper I use functional forms and probability distributions that are convenient for my analysis.

2.1 Production

I consider a discrete time, infinite-horizon economy. There is a unit measure of public firms that produce an homogeneous good. They employ only capital and their profits are assumed to be non-zero and equal to

$$V_{it} = A^{\omega_{it}} K_{it} - \frac{1}{2} K_{it}^2 \tag{1}$$

where

$$\omega_{it} = \begin{cases} l & \mathbb{P} = \frac{1}{2} \\ h & \mathbb{P} = \frac{1}{2} \end{cases}$$

and $0 < A^l < A^h$. The average productivity level is $\bar{A} \equiv \frac{1}{2}\left(A^h + A^l\right)$. I assume ω_{it} to be i.i.d. across firms. The only source of uncertainty in the economy consists in the idiosyncratic risk associated to $A^{\omega_{it}}$. Aggregate shocks are perfectly observed by all economic agents and their effect consists in modifying the expected value or the dispersion of the random variable $A^{\omega_{it}}$. There are two types of them: an aggregate productivity shock and an uncertainty shock. They are defined as follows:

Definition 1. *A positive (negative) aggregate productivity shock increases (decreases) the average productivity level $\bar{A}$, leaving the dispersion $A^h - A^l$ unaffected, i.e. $\Delta A^h = \Delta A^l > 0$ if the aggregate productivity shock is positive and $\Delta A^h = \Delta A^l < 0$ if it is negative.*

Definition 2. *An uncertainty (certainty) shock decreases (increases) the dispersion $A^h - A^l$ leaving the average productivity unaffected, i.e. $\Delta A^h = -\Delta A^l$ with $\Delta A^h < 0$ for the certainty shock and with $\Delta A^h > 0$ for the uncertainty shock.*

The first shock captures the idea that in booms productivities in the high and in the low state are higher than in busts. The second shock instead represents the idea that there are periods when even though the productivity level is still not perfectly observed the informational friction is moderate (small $A^h - A^l$) and periods when the opposite is true (large $A^h - A^l$).

The problem of the firm is then to choose the level of capital to install under imperfect information of their productivity level. In addition to knowing the prior distribution of $A^{\omega_{it}}$ and observing the aggregate shocks, firms also look at their own stock price to learn about ω_{it} as this is (potentially) informative, as I will show later on. Therefore, the information set of a firm i at time t consists in: the prior distribution of $A^{\omega_{it}}$, the current realizations of the aggregate shocks and her stock price P_{it}. Then, the profit-maximizing level of capital is

$$K_{it} = \mathbb{E}\left[A^{\omega_{it}} \mid I_{it}\right]$$

where I_{it} is firm i's information set at time t.

As it will be clear in Section 4, because firms are ex-ante identical and the stock price is either fully revelatory of ω_{it} or not informative at all, aggregate output can be rewritten as

$$Y_{it} = \mathbb{P}\left(\text{“informative } P_{it}\text{”} \cap \omega_{it} = h\right) \cdot \frac{\left(A^h\right)^2}{2} + \mathbb{P}\left(\text{“informative } P_{it}\text{”} \cap \omega_{it} = l\right) \cdot \frac{\left(A^l\right)^2}{2} + \tag{2}$$

$$\mathbb{P}\left(\text{“uninformative } P_{it}\text{”}\right) \cdot \frac{\left(\bar{A}\right)^2}{2} \tag{3}$$

where $\frac{\left(A^h\right)^2}{2}$, $\frac{\left(A^l\right)^2}{2}$ and $\frac{\left(\bar{A}\right)^2}{2}$ are the output levels of, respectively, a fully-informed high-productivity firm, of a fully-informed low-productivity firm and of an “ignorant” firm. All these three probabilities are endogenous to the decisions of speculators and firms and will be affected by aggregate shocks. I pass then to outlining the environment of the financial market.

2.2 Financial market

The firms' stocks are traded in a secondary market, i.e. no capital is flowing to firms, whose micro-structure is in the spirit of Kyle (1985). There are three types of agents: a unit measure of speculators[1], a unit measure of noise traders and a market-maker. The stocks of each firm are traded by one and only speculator such that for every firm there is a monopolistic trader and by one and only noise trader[2]. Importantly, both are subject to position limits: they can trade (buy or sell) up to δ units of the stock of the firm on which they are allowed to invest.

The monopolistic speculator can acquire a signal s_{it} about the state of the world ω_{it} of the firm on which she invests. The signal takes the form

$$s_{it} = \begin{cases} \omega_{it} & \mathbb{P} = \lambda \\ \varnothing & \mathbb{P} = 1 - \lambda \end{cases}$$

i.e. with a frequency λ it reveals the firm-specific state of the world and otherwise it is completely uninformative. I refer to λ as the level of “information precision” chosen by a trader. However, because the signal is either perfectly informative or uninformative, the correct interpretation of λ is the frequency at which the trader wants to be perfectly informed about ω_{it}. The cost of information is assumed to be convex so that acquiring more “precise” information becomes increasingly costlier: $c(\lambda) = \lambda^2$. After observing the signal, the monopolistic trader demands $x_{it} \in [-\delta, \delta]$ stocks of firm i,

[1]The terms “speculator” and “trader” are used interchangeably.

[2]I assume that there is a monopolistic speculator to avoid our focus to be carried away by the insurgence of several equilibria due to the interactions between several traders.

i.e. $x_{it} = x_{it}(s_{it})$.

The noise trader demands stocks according to a random variable Δ_{it} that can take take value $-\delta$ or δ with equal probability. The presence of noise traders ensures that equilibrium prices do not fully reveal the private information of speculators that otherwise would have no trading profits and so would not be willing to acquire private information (Grossman and Stiglitz, 1980). Then the total order flow for a firm i in period t is

$$Q_{it} = x_{it} + \Delta_{it}$$

The market-maker observes the total order flow Q_{it} and sets the price of firm i's stock to break even in expectation: $P_{it} = \mathbb{E}[V_{it} \mid Q_{it}]$ ("fair price").

3 The formation of the equilibrium price

In order to understand how the private information of traders flows into the stock price and so is a valuable signal of ω_{it} for firm i that decides on capital under imperfect information, I derive the equilibrium price. Because the pricing rule is based on Q_{it}, when setting it up the market-maker needs to take into account what information Q_{it} reveals to the firm. In other words, the market-maker's pricing rule and the firm's decision on capital are determined jointly and indirectly depend on the trading strategy of the monopolistic speculator as this determines the information contained in Q_{it}. Therefore to determine how the private information of traders is incorporated in the stock price, one needs to figure out how the monopolistic trader, the market-maker and the firm behave in equilibrium. This the outcome of a game between these three agents. I call it the "pricing-installing-trading" (PIT in short) game because it will consist of a pricing rule, a strategy specifying the level of capital to be installed and a trading strategy. Before introducing it, I make clear the sequence of events concerning firm i and the associated monopolistic trader.

Timing. The timing is as follows:

(i) the monopolistic trader decides the level of information precision λ;

(ii) having observed s_{it}, she sends an order x_{it}; at the same time noise traders send a total order according to the realization of Δ_{it};

(iii) the market-maker sets the price P_{it} given the total order flow Q_{it};

(iv) conditional on P_{it}, firm i decides on the level of capital K_{it} to be installed ;

(v) the firm-specific state of the world ω_{it} realizes and positions in the financial market are cleared.

In next paragraphs, I find an equilibrium of the PIT game and derive the associated expected trading profits for a given information precision λ. Then, I compute the optimal λ. Before going through these steps, I make clear a point on the equilibrium of the PIT game.

3.1 Preliminary results

Because the speculator is monopolistic, she wants to keep trading activity as less informative as possible so to appropriate the rent represented by the difference between the actual value of the firm (that depends on the productivity level on which she has private information) and the market-maker's valuation of the firm (which is based on the total level of trading activity, hence indirectly also on her strategy). Since it is common-knowledge that the order from the noise trader is either δ or $-\delta$ and the speculator and the noise trader are subject to the same position limits, whenever the total order flow Q_{it} is different from $\delta, -\delta$ or 0 the market-maker perfectly observes the order of the monopolistic trader and so can infer her private information. As it will be clear later on, the equilibrium strategy of the monopolistic trader is buying δ units of the stock when she receives a positive signal, selling δ units when she receives a negative signal and not trading when the signal is empty.

Observing the total order flow, one can infer that $x_{it} = \delta$ and so $s_{it} = h$ when $Q_{it} = 2\delta$, that $x_{it} = -\delta$ and so $s_{it} = l$ when $Q_{it} = -2\delta$ and that $x_{it} = 0$ and so $s_{it} = \varnothing$ when $Q_{it} \in \{-\delta, 0, \delta\}$. Therefore, based on the information contained in Q_{it}, the optimal decision rule for the firm is choosing A^h when $Q_{it} = 2\delta$, A^l when $Q_{it} = -2\delta$ and $\bar{A}$ when $Q_{it} \in \{-\delta, 0, \delta\}$.

In the next section I show that there is an equilibrium in the PIT game where the market-maker's strategy reveals the information carried by the stock price and so allows the firm to learn her "type" ω_{it}, whenever Q_{it} is informative. In this equilibrium, the stock market alleviates the informational friction and so plays an informational role. Note that although there is also an "uninformative equilibrium", the"informative one" is the unique equilibrium if one assumes that the firm can observe Q_{it}.

3.2 Equilibrium

The equilibrium concept I employ is Perfect Bayesian Equilibrium. An equilibrium of the PIT game is defined as follows.

Definition 3 *An equilibrium of the PIT game consists of a profile of strategies* $(x_{it}(s_{it}), P_{it}(Q_{it}), K_{it}(P_{it}))$

and a system of beliefs (μ, η) such that (i) the trading strategy $x_{it}(s_{it})$ maximizes the expected profits of the monopolistic speculator, given her signal, the market-maker's pricing rule and the firm's capital decision rule; (ii) the pricing rule $P_{it}(Q_{it})$ makes the market-makers break-even in expectation, given the total order flow and her beliefs μ, the trading strategy of the monopolistic speculator and the firm's capital decision rule; (iii) the capital decision rule maximizes the firm's expected profits, given the price of her stock and her beliefs η, the trading strategy of the monopolistic speculator and the market-maker's pricing rule; (iv) the belief system (μ, η) is derived from the strategy profile $(x_{it}(s_{it}), P_{it}(Q_{it}), K_{it}(P_{it}))$ using Bayes' rule whenever possible.

In the next proposition I outline an equilibrium of the game.

Proposition 1 *The triplet $(x_{it}(s_{it}), P_{it}(Q_{it}), K_{it}(P_{it}))$ such that the monopolistic speculator invests according to*

$$x_{it} = \begin{cases} \delta & \text{if} \quad s_{it} = h \\ -\delta & \text{if} \quad s_{it} = l \\ 0 & \text{if} \quad s_{it} = \varnothing \end{cases} \tag{4}$$

the market-maker prices the stock using the rule

$$P_{it} = \begin{cases} V^h = \frac{(A^h)^2}{2} & \text{if} \quad Q_{it} \in (-\delta, 0) \cup (\delta, 2\delta] \\ V^l = \frac{(A^l)^2}{2} & \text{if} \quad Q_{it} \in [-2\delta, -\delta) \cup (0, \delta) \\ \bar{V}^{\varnothing} = \frac{(\bar{A})^2}{2} & \text{if} \quad Q_{it} \in \{-\delta, 0, \delta\} \end{cases} \tag{5}$$

and the firm chooses capital based on

$$K_{it} = \begin{cases} A^h & \text{if} \quad P_{it} \in (\bar{V}^{\varnothing}, V^h] \\ A^l & \text{if} \quad P_{it} \in [V^l, \bar{V}^{\varnothing}) \\ \bar{A} & \text{if} \quad P_{it} = \bar{V}^{\varnothing} \end{cases} \tag{6}$$

paired with the the market-maker's beliefs

$$\mu = \begin{cases} \mu(\omega_{it} = h \mid Q_{it}) = 1 & \text{if} \quad Q_{it} \in (-\delta, 0) \cup (\delta, 2\delta] \\ \mu(\omega_{it} = l \mid Q_{it}) = 1 & \text{if} \quad Q_{it} \in [-2\delta, -\delta) \cup (0, \delta) \\ \mu(\omega_{it} = h \mid Q_{it}) = \frac{1}{2} & \text{if} \quad Q_{it} \in \{-\delta, 0, \delta\} \end{cases} \tag{7}$$

and the firm's beliefs

$$\eta = \begin{cases} \eta(\omega_{it} = h \mid P_{it}) = 1 & \text{if} \quad P_{it} \in (\bar{V}^{\varnothing}, V^h] \\ \eta(\omega_{it} = l \mid P_{it}) = 1 & \text{if} \quad P_{it} \in [V^l, \bar{V}^{\varnothing}) \\ \eta(\omega_{it} = h \mid P_{it}) = \frac{1}{2} & \text{if} \quad P_{it} = \bar{V}^{\varnothing} \end{cases} \tag{8}$$

forms an equilibrium of PIT game.

Proof. First, I check that the trader is choosing her strategy optimally, given her signal, the market-maker's pricing rule and the firm's capital decision rule. Assume $s_{it} = h$, then:

▷ demanding $x_{it} = \delta$ provides an expected payoff equal to $\frac{\delta}{2}\left[A^h \bar{A} - \bar{A}^2\right] > 0$

▷ demanding $0 < x_{it} < \delta$ provides an expected payoff equal to 0

▷ demanding $-\delta \leq x_{it} \leq 0$ provides an expected payoff equal to $\frac{x}{2}\left[\left(A^l\right)^2 - A^h A^l\right] \leq 0$

Then, there is no strictly profitable deviation from $x_{it} = \delta$.

Because the two cases are symmetric, the same logic applies to proving that there is no strictly profitable deviation from $x_{it} = -\delta$ when $s_{it} = l$.

Lastly, when $s_{it} = \varnothing$, the strategies $x_{it} = 0$ and $x_{it} = -\delta$ and $x_{it} = \delta$ are the only ones that provide a zero payoff. All the others imply a loss because: when buying ($x_{it} > 0$) the price is higher then the trader's valuation of the firm, since she did not learn anything; instead, when selling ($x_{it} < 0$) the price is lower then the trader's valuation.

I check that the market-maker behaves optimally, given the total order flow and her beliefs, the trading strategy of the monopolistic speculator and the firm's capital decision rule. As already explained in Section 3.1, on path the market-maker behaves optimally. Off-path, when $Q_{it} \in (-\delta, 0) \cup (\delta, 2\delta)$, the market-maker chooses $P_{it} = \frac{(A^h)^2}{2}$. Because the firm responds with $K_{it} = A^h$ and the market maker believes the firm to be highly productive with probability 1, she indeed breaks-even in expectation and so is in the optimum. When $Q_{it} \in (-2\delta, -\delta) \cup (0, \delta)$, the market-maker chooses $P_{it} = \frac{(A^l)^2}{2}$. Because the firm, in response, installs $K_{it} = A^l$ and the market-maker believes the firm's productivity to be low with probability 1, she indeed breaks-even in expectation. Therefore, again, she is in the optimum. The belief μ is derived using Bayes'rule when the information set is reached on path.

It is only left to check that the firm is choosing the optimal strategy, given the price and her beliefs, the trading strategy of the monopolistic speculator and the market-maker's pricing rule. As explained in Section 3.1, on path the firm behaves optimally. Off-path, when $P_{it} \in (\bar{V}^{\varnothing}, V_h^h)$ she believes to be in the high state and so it is optimal to choose A^h. When $P_{it} \in (V_l^l, \bar{V}^{\varnothing})$, she believes to be in the low state and so it is optimal to choose A^l. The belief η is derived using Bayes'rule when the information set is reached on path.

Because the strategy profile $(x_{it}(s_{it}), P_{it}(Q_{it}), K_{it}(P_{it}))$ is sequentially rational given the belief system (μ, η) and the belief system (μ, η) is derived using Bayes' rule whenever possible, it forms an equilibrium of the PIT game. □

Note that there is also another equilibrium where the market-maker's strategy does not communicate any information to the firm. This consists in the market-maker setting the same price for any observed total order flow, i.e. $P_{it} = \bar{V}^{\varnothing}\ \forall\ Q_{it}$, the firm choosing the average productivity level to be her capital, i.e. $K_{it} = \bar{A}\ \forall\ P_{it}$, and the trader buying the maximum amount possible δ if $s_{it} = h$ and selling it if $s_{it} = l$. However, the equilibrium of Proposition 1 is the only one if I assume that the firm can observe the total order flow[3].

3.3 Information acquisition

In order to decide on the level of information precision, the monopolistic trader maximizes her expected trading profits for an given λ net of the corresponding cost, i.e.

$$\max_{\lambda} \pi(\lambda) - c(\lambda) \tag{9}$$

Lemma 1. *Given the trading strategy in (4), the pricing rule in (5) and the firm's capital decision*

[3]In this case, Q_{it} is a sufficient statistic for the price, which therefore becomes redundant. The equilibrium in Proposition 1 could be obtained in a game without the market-maker. However, it is useful to define the equilibrium price to allow the model to be empirically assessed as this is the variable that is observed in the data rather than the total order flow.

in (6), the monopolistic trader earns expected profits equal to

$$\pi(\lambda) = \frac{\lambda}{4} \cdot \bar{A}\left(A^h - A^l\right)$$

Proof. With probability λ the monopolistic trader receives a signal. If $s_{it} = h$, she buys δ units of the stock. With probability $\frac{1}{2}$ the total order flow perfectly reveals the signal ($Q_{it} = 2\delta$) and trading profits are zero. With equal probability, instead, the total order flow is completely uninformative ($Q_{it} = 0$) and trading profits are $V_h^{\varnothing} - \bar{V} = \frac{1}{2} \cdot \bar{A}\left(A^h - A^l\right)$. Summing up, if $s_{it} = h$, trading profits are

$$\left[\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot \bar{A}\left(\frac{A^h - A^l}{2}\right)\right]$$

If $s_{it} = l$, trading profits are identical as above, whereas if $s_{it} = \varnothing$ they are zero. Putting everything together, expected profits are

$$\frac{\lambda}{4} \cdot \bar{A}\left(A^h - A^l\right)$$

□

One comment needs to be in place to orient the interpretation of the comparative statics on the parameters of $\pi(\lambda)$. Because the trading activity is either fully revealing of the speculator's private information or not informative at all, the speculator's profits are strictly positive only when Q_{it} is uninformative. In this instance, the stock is priced at $P_{it} = \frac{(\bar{A})^2}{2}$ and the firm installs $K_{it} = \bar{A}$. Observing the true idiosyncratic state of the world, a trader knows that the firm's profits will be $V_h^{\varnothing} = A^h\bar{A} - \frac{(\bar{A})^2}{2}$ if $\omega_{it} = h$ and so her trading profits are $V_h^{\varnothing} - \bar{V} = \frac{1}{2} \cdot \bar{A}\left(A^h - A^l\right)$ as shown in the proof above; seemingly, if $\omega_{it} = h$ her trading profits are $\bar{V} - V_l^{\varnothing} = \frac{1}{2} \cdot \bar{A}\left(A^h - A^l\right)$.

Then expected trading profits $\pi(\lambda)$ are increasing in the average productivity level because a higher $\bar{A}$ implies that the firm installs more capital and so its profits are more exposed to the realization of the firm-specific state of the world ω_{it}, i.e. $V_h^{\varnothing} - V_l^{\varnothing}$ grows. As a result, trading profits increase because the gap between the actual value of the firm (which the speculator observes perfectly) and the price widens. Similarly, for a given level of capital, more dispersed productivity levels, i.e. a bigger $A^h - A^l$, imply that profits fluctuate more between $\omega_{it} = h$ and $\omega_{it} = l$ and so $\pi(\lambda)$ is also increasing in $A^h - A^l$. Lastly, a higher λ increases the frequency at which the monopolistic trader perfectly learn the true idiosyncratic state of the world which implies that more often she identifies the mispricing.

Having said this, the level of information precision that solves (9) when $c(\lambda) = \lambda^2$ is

$$\lambda^\star = \frac{1}{8} \cdot \bar{A}\left(A^h - A^l\right)$$

4 The aggregate economy and the amplification mechanism

Until now I have considered the trade of the stocks of an arbitrary firm i. Because firms are ex-ante identical (they all maximize profits in (1) under the same information set), the results derived in the previous sections apply to any of them. This, coped with the fact that the signal s_{it} is i.i.d across firms, allows me to apply a law of large numbers to write aggregate output. As previously anticipated, a fraction $\frac{1}{4}\lambda$ of firms learns from their own stock price that $\omega_{it} = h$, an equal fraction that $\omega_{it} = l$ and the remaining fraction $\frac{2-\lambda}{2}$ does not learn anything. Then, aggregate output is

$$Y_{it} = \frac{1}{4}\lambda \cdot \frac{\left(A^h\right)^2}{2} + \frac{1}{4}\lambda \cdot \frac{\left(A^l\right)^2}{2} + \frac{2-\lambda}{2} \cdot \frac{\left(\bar{A}\right)^2}{2}$$

which can rewritten as

$$Y_{it} = Y_{it}^P - LOSS_{it}$$

where Y_{it}^P is aggregate output under perfect information which is equal to $\frac{1}{2} \cdot \frac{\left(A^h\right)^2}{2} + \frac{1}{2} \cdot \frac{\left(A^l\right)^2}{2}$ and $LOSS_{it}$ is the output loss due to imperfect information. This is given by the fact that a fraction of firms equal to $\frac{1}{4}\lambda$ has a high productivity level but does not learn anything from the stock price because of the “hide behind the noise trader” strategy of the monopolistic speculator. These firms install $K_{it} = \bar{A}$ while under perfect information they would have chosen A^h. The same happens for another $\frac{1}{4}\lambda$ fraction of firms that have a low productivity level and does not learn from the stock price. Lastly, a fraction $1 - \lambda$ does not learn her productivity level because not even the monopolistic trader did. Half of this fraction is made of firms that have a high productivity and the other half of low-productivity ones.

Therefore the output loss takes the form $LOSS_{it} = \frac{1}{4}\lambda \cdot \alpha + \frac{1}{4}\lambda \cdot \beta + (1-\lambda)\left[\frac{1}{2}\alpha + \frac{1}{2}\beta\right]$ where

$$\alpha = \frac{\left(A^h\right)^2}{2} - \left[A^h \bar{A} - \frac{\left(A^h\right)^2}{2}\right]$$

is the output loss due to "ignorant" high productivity firms and

$$\beta = \frac{\left(A^l\right)^2}{2} - \left[A^l \bar{A} - \frac{\left(A^l\right)^2}{2}\right]$$

is the output loss due to "ignorant" low productivity firms.

The effect of more precise information acquired by monopolistic traders, i.e. a higher λ, is that a larger fraction of firms perfectly learns about their own productivity level and so chooses capital under perfect information. In the terminology used in the introduction, when traders acquire more precise signals, the stock price becomes more informative, input misallocation decreases and output rises. So a higher λ is associated with greater information production in the stock market. Note that, even when $\lambda = 1$, the informational friction is not removed, however. This is precisely because of the profit-seeking motif of the monopolistic speculators that try to keep their trading activity as less informative as possible. Indeed, when $\lambda = 1$, traders are always privately informed about the productivity level of the firm and still $\frac{1}{4}$ of high-productivity firms and $\frac{1}{4}$ of low-productivity ones keeps being ignorant about ω_{it}.

4.1 Amplification mechanism

In this section I show that the response of the economy to any of the aggregate shocks previously defined is amplified by changes in the level of information precision acquired by traders. The main idea behind the result is that these shocks affect the mispricing of the stock and so the size of the rents that traders can appropriate by acquiring a more precise signal.

Proposition 3. *The endogenous response of information precision λ to an aggregate productivity shock amplifies its direct effect on aggregate output, i.e.*

$$\frac{dY_{it}}{d\bar{A}} = \frac{\partial Y_{it}}{\partial \bar{A}} + \frac{\partial Y_{it}}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial \bar{A}} > \frac{\partial Y_{it}}{\partial \bar{A}} > 0$$

Proof. Because the endogenous response of information precision λ to an aggregate productivity shock is

$$\frac{\partial \lambda}{\partial \bar{A}} = \frac{1}{8}\left(A^h - A^l\right) > 0$$

and the response of aggregate output to a change in the information precision is

$$\frac{\partial Y_{it}}{\partial \lambda} = \frac{\left(A^h\right)^2 + \left(A^l\right)^2 + 2\left(\bar{A}\right)^2}{8} > 0 \tag{10}$$

the effect of change in information precision on aggregate output is positive (indirect effect). The direct effect is

$$\frac{\partial Y_{it}}{\partial \bar{A}} = (1+\lambda)\,\bar{A} > 0$$

□

The amplification mechanism works as follows. When a positive aggregate productivity shock hits the economy, all firms (those that do not learn anything in equilibrium and those that perfectly learn their state) install more capital because their productivity level (either their true one or the perceived one, i.e. $\bar{A}$) has increased. This is the direct effect. In particular, as firms grow in size, their profits are more exposed to the realization of the firm-specific state of the world, i.e. $V_h^{\varnothing} - V_l^{\varnothing}$ increases. Because the market maker prices stocks at the average firms' profit, knowing whether $\omega_{it} = h$ or $\omega_{it} = l$ now provides a higher rent to the monopolistic trader because the market-maker misprices the stock more heavily. The equilibrium level of information precision $\lambda^\star$ increases for every firm and a larger fraction of them learns their true productivity level. The severity of the informational friction is reduced and aggregate output is closer to the full information case where the productivity level is perfectly observed (indirect effect).

Proposition 4. *The endogenous response of information precision λ to an uncertainty shock amplifies its direct effect on aggregate output.*

Proof. The direct effect is

$$\frac{\partial Y_{it}}{\partial\left(A^h - A^l\right)} = \frac{1}{4}\left(A^h - A^l\right) > 0$$

The indirect effect is given by (10) which is strictly positive and

$$\frac{\partial \lambda}{\partial \left(A^h - A^l\right)} = \frac{1}{8}\bar{A} > 0$$

This shows that the indirect effect is strictly positive and so changes in λ amplify the direct response of the economy to an uncertainty shock. $\square$

The direct effect of an uncertainty shock on output is positive even though A^h increases as much as A^l decreases, because equilibrium profits are quadratic in the productivity level. The amplification in this case comes through a different channel. Here, firms do not actively contribute to the amplification mechanism by modifying their level of capital in a way that widens the rents of traders. The result is more mechanical. As the dispersion $A^h - A^l$ rises, for an arbitrary K_{it}, the firms' profits fluctuate more between the high and the low state and so the mispricing of the stock automatically grows. The monopolistic speculator acquires more precise information and the severity of the informational friction is reduced.

5 Conclusion

Stock markets play a fundamental role in the economy because they provide information that helps firms allocating factors to production and so reduces input misallocation due to imperfect information. In this paper, I study the impact of aggregate shocks on information production in the stock market and how its response feedbacks into the aggregate efficiency of the economy and output. This provides a novel amplification mechanism linking together firms' imperfect information and the informational role of stock markets. I show that the business cycle affects the incentives of speculators to produce information: in expansions, as firms grow, the mispricing of the stock increases and so traders acquire more precise information to capture the larger rent; in periods of greater uncertainty, the same is true. As a result, changes in information production amplify an aggregate shock.

In the future, I plan to extend the results covered in this paper in several directions. First, firms choose inputs in partial equilibrium. The consequence is that I can identify the overall output loss due to imperfect information and how this is affected by changes in information production, but it is complicated to separately identify the effect on the aggregate efficiency distorsion and that on aggregate investment as such things do not exists in partial equilibrium. Therefore, my goal is to use the mechanism presented here to model the interaction between the stock market and the aggregate economy in a general equilibrium framework. This will also allow me to assess the relevance of the informative channel through which shocks are amplified.

Secondly, I would like to estimate the extent to which the business cycle alleviates the informational friction due to imperfect information. Under perfect information, input choices perfectly comove with the firm-specific productivity level, whereas under imperfect information this interaction is crucially mediated by stock prices. Therefore, estimating the extent to which stock prices comove with the underlying firm productivity and aggregate investment comoves with stock prices should give a sense of how far the economy is from the perfect information case at a given point in time. Since in the model the gap between the perfect and imperfect information economy depends on the level of information production in the stock market, this allows me to estimate the informational role of the stock market at business cycle frequencies and by extension how the business cycle affects firm-level uncertainty.

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